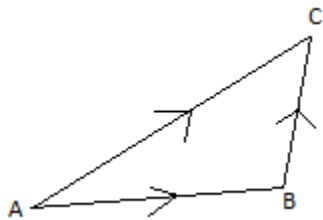


# Vector

- Addition of vectors** : If  $a = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$  and  $b = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ ,  $a + b = \begin{pmatrix} x_1+x_2 \\ y_1+y_2 \end{pmatrix}$
- Multiplication of vectors** :  $k \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx \\ ky \end{pmatrix}$
- Modulus (magnitude) of a vector**: If  $a = \begin{pmatrix} x \\ y \end{pmatrix}$  modulus (magnitude) of  $a = \sqrt{x^2 + y^2}$
- Angle of a vector**: If  $a = \begin{pmatrix} x \\ y \end{pmatrix}$  and angle between  $a$  and x-axis is  $\theta$ ,  $\tan\theta = \frac{y}{x}$ .
- Parallel vectors**: (i) If  $a = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$  and  $b = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$  are parallel,  $\frac{x_1}{y_1} = \frac{x_2}{y_2}$   
 (ii) If  $(ha + kb) \parallel (ma + nb)$ ,  $\frac{h}{k} = \frac{m}{n}$
- Equal vectors**: (i) If  $a = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$  and  $b = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$  are equal,  $x_1 = x_2$  and  $y_1 = y_2$   
 (ii) If  $(ha + kb) = (ma + nb)$ ,  $h=m$  and  $k=n$
- Coordinates of A means  $\vec{OA}$
- $\vec{AB} + \vec{BC} = \vec{AC}$



- If two lines are equal and parallel, their vectors are equal.
- If  $\vec{AB} = a$ ,  $\vec{BA} = -a$
- Vector can be found by the following three methods:
  - Route method:  $\vec{AC} = \vec{AB} + \vec{BC}$ ,  $\vec{AD} = \vec{AB} + \vec{BC} + \vec{CD}$ ,  $\vec{AE} = \vec{AB} + \vec{BC} + \vec{CD} + \vec{DE}$  etc.
  - Ratio method: wanted vector =  $\frac{\text{wanted vector ratio}}{\text{known vector ratio}}$  x known vector.
  - Geometrical method: If  $AB \parallel CD$  and  $AB = CD$ ,  $\vec{AB} = \vec{CD}$   
 If  $AB \parallel CD$  and  $AB = kCD$ ,  $\vec{AB} = k\vec{CD}$
- If C is the **mid point** of AB,  $\vec{AC} = \frac{1}{2}\vec{AB}$
- If you need to find AB:CD, divide  $\vec{AB}$  by  $\vec{CD}$ . If the result is  $\frac{m}{n}$ , where m and n are integers, AB:CD = m:n.
- If  $a=kb$ , a and b are parallel.
- If you need to find two geometrical properties between AB and CD, divide  $\vec{AB}$  by  $\vec{CD}$ . If the result is  $\frac{m}{n}$ , where m and n are integers, two geometrical properties are:

- (i)  $AB \parallel CD$
  - (ii)  $AB:CD = m:n$
16. If you need to prove that A, B and C lie on a straight line (collinear), follow the following steps.
- (i) Find two vectors with these three points so that there is a common point between them. The vectors may be  $\overrightarrow{AB}, \overrightarrow{AC}$  or  $\overrightarrow{BC}, \overrightarrow{AC}$ .
  - (ii) Divide one vector by another.
  - (iii) If the result is a constant (only number), this proves that A, B and C lie on a straight line.
17. (i) If  $\overrightarrow{AB} = \overrightarrow{DC}$  or  $\overrightarrow{AD} = \overrightarrow{BC}$ , ABCD is a parallelogram.
- (ii) If ABCD is a parallelogram,  $\overrightarrow{AB} = \overrightarrow{DC}$  or  $\overrightarrow{AD} = \overrightarrow{BC}$
18. Finding area ratio: If two triangles are similar, their area ratio is equal to square of side ratio.

# Function

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1. A function can be written in the following 3 forms:
  - (i)  $y = 5x - 2$
  - (ii)  $f(x) = 5x - 2$
  - (iii)  $f: x \rightarrow 5x - 2$
2. When a function is written in arrow form, first letter is the name of the function, the letter between colon and arrow sign is domain ( $x$ ), the part after the arrow is range ( $y$ ).
3. To find  $f(2)$ , replace  $x$  of  $f$  function by 2.
4. **Composite function:**
  - (i) To find  $fg$  or  $fg(x)$ , replace  $x$  of  $f$  function by  $g$  or  $g(x)$  function. Except  $x$ , everything of  $f$  function will remain same.
  - (ii) To find  $fg(2)$ , first find  $g(2)$  then find  $f(g(2))$ .
5. If original function is  $f$ , inverse function is  $f^{-1}$ .
6. To find inverse of  $f$ , follow the following steps:
  - (i) Write  $f$  function in  $x, y$  form.
  - (ii) Change  $x$  by  $y$  and  $y$  by  $x$ .
  - (iii) Make  $y$  subject.
  - (iv) Write  $f^{-1}$  in the wanted form.
7. If in  $f$  function,  $x > a$  and  $y > b$ , in  $f^{-1}$  function  $x > b$  and  $y > a$ .
8. In fraction function, the value of  $x$  for which the denominator becomes zero, must be excluded.
9.  $x$  values of a function are called domain and  $y$  values are called range.
10. To find the range of a quadratic function,
  - (i) Find the maximum or minimum values of  $y$ .
  - (ii) If  $y$  is maximum range is:  $y \leq \text{maximum value}$
  - (iii) If  $y$  is minimum, range is:  $y \geq \text{minimum value}$
11. To find the range of any function
  - (i) Draw a sketch of the function for the given domain.
  - (ii) Find the range from the sketch.

# Matrix

1. **Multiplication by a scalar** :  $k \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$
2. **Addition of matrix** :  $\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} a+w & b+x \\ c+y & d+z \end{pmatrix}$
3. **Subtraction of matrix** :  $\begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} a-w & b-x \\ c-y & d-z \end{pmatrix}$
4. Two matrix can be added or subtracted if their number of row and column are equal.
5. **Multiplication of two matrix** :  $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} aw+by & ax+bz \\ cw+dy & cx+dz \end{pmatrix}$
6. The order of a matrix is (2 x 3) means it has 2 row and 3 column.
7. If two matrices have order (p x q) and (r x s), they can be multiplied if q=r and the order of the resultant matrix will be (p x s)
8. **Equal matrix** : If  $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} w & x \\ y & z \end{pmatrix}$ ,  $a = w, b = x, c = y, d = z$
9. Determinant of  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$  is  $ad - bc$ .
10. Inverse of  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$  is  $\frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$
11. If  $ad - bc = 0$ , the matrix has no inverse.
12. (i) (2 x 2) identity matrix  $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$   
 (ii) (3 x 3) identity matrix  $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
13. If any matrix is multiplied by identity matrix, the matrix remains same.
14. The product of inverse matrix and original is identity matrix.
15. **Reflection:**
  - (i) **Finding image when object and reflection line is given :**
    - (a) If reflection line is horizontal or vertical, count the distance of the mirror line from the object. Image will be on the opposite side of the mirror line with the same distance.
    - (b) If reflection line is slant, move horizontally from the object to meet the mirror line and count the distance. Move vertically from the meeting point with the same distance. Remember that object and image must lie on the opposite side of the reflection line.
  - (ii) **To find mirror line (reflection line) when object and image are given, follow the following steps.**
    - (a) Join any corresponding object and image.
    - (b) Draw the perpendicular bisector of this joining line.
    - (c) This perpendicular bisector is the mirror line.
16. **Rotation.**
  - (i) **Rules of changing points in different rotation :**

- (a)  $\begin{pmatrix} x \\ y \end{pmatrix} \xrightarrow{90^\circ \text{ clockwise}} \begin{pmatrix} y \\ -x \end{pmatrix}$
- (b)  $\begin{pmatrix} x \\ y \end{pmatrix} \xrightarrow{90^\circ \text{ anticlockwise}} \begin{pmatrix} -y \\ x \end{pmatrix}$
- (c)  $\begin{pmatrix} x \\ y \end{pmatrix} \xrightarrow{180^\circ} \begin{pmatrix} -x \\ -y \end{pmatrix}$

**(ii) Finding image when object, centre, angle and direction are given :**

- (a) Write the distance of the object from the centre in matrix form
- (b) If centre is origin, distance from the centre and coordinates of the points will be same, otherwise it will be different.
- (c) For  $90^\circ$  clockwise rotation, change the position of the number and change the sign of the lower number.
- (d) For  $90^\circ$  anticlockwise rotation, change the position of the number and change the sign of the upper number.
- (e) For  $180^\circ$  rotation, do not change the position of any number, but change the sign of all numbers

**(iii) When object and image are given, to find centre, angle and direction follow the following steps.**

- (a) Join any two pairs of corresponding object and image.
- (b) Draw the two perpendicular bisector of these two joining line.
- (c) Intersecting point of these two perpendicular bisector is the centre of the rotation.
- (d) To find angle, join centre and one pair of corresponding object and image. The angle between these two joining lines is the angle of rotation. Measure this angle using protractor, or calculate this angle making triangle.
- (e) To find direction, take a compass. Put the compass leg on the centre and pencil on the object. Keeping the compass leg fixed, move the pencil to the corresponding image. If the pencil moves clockwise, direction is clockwise. If the pencil moves anticlockwise, direction is anticlockwise.

**17. Translation:**

- (i) When object and image are given to find vector, write the coordinates of any corresponding object and image in vector form and subtract object from image.

$$\text{Vector} = \text{Image} - \text{Object}$$

- (ii) When object and vector are given to find image, write the coordinates of the object in matrix form and add this matrix with the given vector.

$$\text{Image} = \text{Object} + \text{Vector}$$

If number of column of the object matrix is 3, write the vector three times, if number of column

of the object matrix is 4, write the vector four times. If object is  $\begin{pmatrix} a & c & e \\ b & d & f \end{pmatrix}$  and vector is  $\begin{pmatrix} g \\ h \end{pmatrix}$ , image will be

$$\begin{pmatrix} a & c & e \\ b & d & f \end{pmatrix} + \begin{pmatrix} g & g & g \\ h & h & h \end{pmatrix} = \begin{pmatrix} a+g & c+g & e+g \\ b+h & d+h & f+g \end{pmatrix}$$

- (iii) When image and vector are given, to find object, write the coordinates of the image in matrix form and subtract vector from this matrix.

Object = image = vector

If number of column of the image matrix is 3, write the vector three times, if number of column of the image matrix is 4, write the vector four times. If image is  $\begin{pmatrix} a & c & e \\ b & d & f \end{pmatrix}$  and vector is  $\begin{pmatrix} g \\ h \end{pmatrix}$ , image will be

$$\begin{pmatrix} a & c & e \\ b & d & f \end{pmatrix} - \begin{pmatrix} g & g & g \\ h & h & h \end{pmatrix} = \begin{pmatrix} a-g & c-g & e-g \\ b-h & d-h & f-g \end{pmatrix}$$

### 18. Enlargement :

**(i) When object, scale factor and centre are given, to find image follow the following steps.**

- Write the distance of the object from the centre in matrix form
- If centre is origin, distance from the centre and coordinates of the points will be same, otherwise it will be different.
- Multiply the object matrix by the scale factor to get the image, Image = Scale Factor x Object

**(ii) When object and image are given, to find the centre and scale factor follow the following steps.**

- Join all corresponding objects and image. The intersecting of the joining lines is the centre.
- If the joining lines do not intersect, produce them to a suitable direction. The intersecting point of the produced lines is the centre.
- To find scale factor, divide corresponding image side by object side. If it is difficult to find side length, divide horizontal or vertical distance from centre of image by horizontal or vertical distance of corresponding object.

$$\text{Scale factor} = \frac{\text{Image}}{\text{Object}}$$

**(iii) When object and image lie on the opposite side of the centre of enlargement, scale factor is negative. When object and image lie on the same side of the centre of enlargement, scale factor is positive.**

19. Finding area ratio of object and image: If you need to find the area ratio of object and image, follow any one of the following methods.

- If image is obtained multiplying object by a matrix, find the determinant of that matrix.

$$\text{area of object} : \text{area of image} = 1 : \text{determinant}$$

- Object and image are similar. So similarity formula also can be applied in this case. If side ratio of object and image is  $I_1 : I_2$ ,

$$\text{area of object} : \text{area of image} = (I_1)^2 : (I_2)^2$$

- If scale factor of the enlargement is k, [If k is negative, consider only number]

$$\text{area of object} : \text{area of image} = 1 : k^2$$

- If possible, find area of the object and area of the image by using the formula  $\frac{1}{2} \times \text{base} \times \text{height}$  or  $\frac{1}{2} ab \sin \theta$  if they are triangles. Then find their ratio.

20. When matrix is given, to find image, write the coordinates of object in matrix form and multiply this object matrix by the given matrix.  
Image = Matrix x Object
21. When object and image are known, transformation may be known or unknown, and you need to find the matrix, take the unknown matrix as  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$  and use the following rules.  
 $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times object = image.$
22. **Base vector:** When transformation is known, object and image may be known or unknown, and you need to find the matrix, follow the following steps.  
(i) Draw the x-axis and y-axis.  
(ii) Plot the points I(1, 0) and J(0,1).  
(iii) Find I' (image of I) and J' (image of J) according to the known transformation.  
(iv) If coordinates of I' = (a, c) and J' = (b, d), wanted matrix will be  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ , I' is the 1<sup>st</sup> column and J' is the 2<sup>nd</sup> column of the wanted matrix.
23. **Base vector:** When only matrix is known, object and image are unknown and you need to find the transformation represented by the matrix, follow the following steps.  
(i) Draw x-axis and y-axis.  
(ii) Plot the points: I(1,0) and J(0,1)  
(iii) Plot the points I' (1<sup>st</sup> column of the given matrix) and J' (2<sup>nd</sup> column of the given matrix).  
(iv) Find the transformation between I and I', J and J'.  
(v) If both transformation are same that will be the answer.
24. If  $A \times object = image$ ,  $A^{-1} \times image = object$
25. Some matrices and their transformation are given below:

Matrix	Transformation
1. $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	Rotation of 90° anticlockwise, centre (0,0)
2. $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	Rotation of 90° clockwise, centre (0,0)
3. $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	Rotation of 180°, centre (0,0)
4. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	Reflection in x-axis
5. $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	Reflection in y-axis
6. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	Reflection in y=x
7. $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	Reflection in y=-x
8. $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$	Enlargement, scale factor k, centre (0,0)

26. Identification of transformation and their fully description

Transformation	Identification	Need to describe fully
Enlargement	Change of area or change of side-length.	Centre and scale factor.
Translation	Change of position without rotation.	Vector.

Reflection	Corresponding object-image joining lines are parallel.	Equation of the reflection line.
$180^{\circ}$ rotation	Corresponding object-image joining lines intersect at the same point, which is the centre of $180^{\circ}$ rotation.	Centre
Other rotation	The remaining are other rotation.	Angle, direction and centre.

27.  $180^{\circ}$  rotation and enlargement with scale factor -1 are same.
28. Invariant point: Normally when points are multiplied by matrix, they are changed. But there are some point, after multiplication by a matrix, they remain unchanged. These points are called invariant point.
29. Invariant line: By joining the invariant points, a line may be obtained. This line is called invariant line. If matrix is  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$  and invariant point is  $\begin{pmatrix} x \\ y \end{pmatrix}$ , use  $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$  to find the equation of invariant line.



# Probability

$s = \text{number of A}, t = \text{number of B}, u = \text{number of C}, n = \text{total number}$

1.  $p(A) = \frac{s}{n}, p(B) = \frac{t}{n}, p(C) = \frac{u}{n}$
2. (i)  $p(AA) = \frac{s}{n} \times \frac{s}{n}$  [If replaced]  
 (ii)  $p(AA) = \frac{s}{n} \times \frac{s-1}{n-1}$  [If not replaced]
3. (i)  $p(AB) = \frac{s}{n} \times \frac{t}{n} \times 2$  [If replaced]  
 (ii)  $p(AB) = \frac{s}{n} \times \frac{t}{n-1} \times 2$
4. A, B and C can be arranged in the following 6 ways: ABC, ACB, BAC, BCA, CAB, CBA.
5. (i)  $p(ABC) = \frac{s}{n} \times \frac{t}{n} \times \frac{u}{n} \times 6$  [If replaced]  
 (ii)  $p(ABC) = \frac{s}{n} \times \frac{t}{n-1} \times \frac{u}{n-2} \times 6$  [If not replaced]
6. For 2 draw,  $p(\text{different colour}) = 1 - p(\text{same colour})$ .
7. **AND** rule:  $p(A \text{ and } B) = p(A) \times p(B)$
8. **OR** rule:  $p(A \text{ or } B) = p(A) + p(B)$
9.  $p(A) + p(B) + p(C) = 1$
10. The probabilities of die, ball, bead, sweet etc may change or not according to replacement. But probabilities of die, coin etc always remain unchanged.
11. If probability is given, you have to use the probability.
12. To draw tree diagram, number of branches = number of given items.  
 number of steps = number of draw.

# Kinematics

1. *time = t, distance/displacement = s/x, acceleration/retardation = a, velocity/speed = v*
2.  $v = \frac{dx}{dt}$  ,  $a = \frac{dv}{dt}$
3. For maximum or minimum value of  $s/x$ ,  $\frac{ds}{dt} = 0$
4. For maximum or minimum value of  $v$ ,  $\frac{dv}{dt} = 0$
5. If the particle comes to rest,  $v = 0$ .
6. If the particle comes to fixed point or fixed level,  $s=0$  or  $x=0$ .
7. Distance travelled in  $t$  second =  $s_t$  [If at  $t=0$ ,  $s = 0$ ]
8. Distance travelled in  $t$  second =  $s_1 - s_0$  [If at  $t=0$ ,  $s \neq 0$ ]
9. Distance travelled in  $t$  th second =  $s_1 - s_{t-1}$
10. Distance travelled between  $t = 3$  and  $t = 7 = s_1 - s_3$  [If there are no value of  $t$  between  $t=3$  and  $t = 7$  for which  $v = 0$ ]
11. Distance travelled between  $t=3$  and  $t=7 = (s_5 - s_3) + (s_7 - s_5)$  [If there is no value  $t=5$  between  $t = 3$  and  $t = 7$  for which  $v = 0$ ].
12. If speed is constant, distance = speed x time.
13. Rules of Differentiation:

$$\frac{d}{dt}(\text{only number}) = 0, \frac{d}{dt}(t) = 1, \frac{d}{dt}(t^2) = 2t, \frac{d}{dt}(t^3) = 3t^2$$

$$\text{General rule: } \frac{d}{dt}(t^n) = nt^{n-1}$$

# Set

**1. Number of some symbols:**

(i)  $\cap$  = intersection, (ii)  $\cup$  = union, (iii)  $C$  = subset, (iv)  $\in$  = “is a member of” or “belongs to” or “element of”, (v)  $\mathcal{E}$  = universal set, (vi)  $A'$  = “A prime” or “complement of A”, (vii)  $n(A)$  = number of elements in set A, (viii)  $\emptyset$  = empty set.

2.  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

3.  $n(A) + n(A') = n(\mathcal{E})$

4. (i) maximum  $n(A \cap B) = n(B)$ , where  $n(A) > n(B)$ .

(ii) minimum  $n(A \cap B) = n(A) + n(B) - n(A \cup B)$ .

5. (i) maximum  $n(A \cup B) = n(A) + n(B)$

(ii) minimum  $n(A \cup B) = n(A)$ , where  $n(A) > n(B)$ .

**6. Number facts:**

(i) **Integer number:** The number which are not decimal or fraction are called integer number. Integer numbers are always whole number. Positive integers are 1, 2, 3, 4, 5....., negative integers are -1, -2, -3, -4, -5..... The number 0 is also considered as integer.

(ii) **Prime number:** A prime number is divisible only by itself and by one. First 20 positive prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71

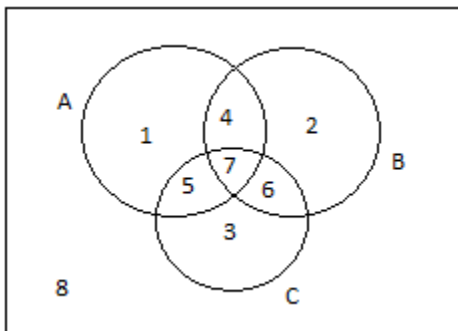
(iii) **Multiples of number:** If y is divisible by x, y is called multiple of x. For example, multiples of 3 are 3, 6, 9, 12, 15, 18, 21.

(iv) **Factors of number:** If y is divisible by x, x is called factor of y. For example, factors of 18 are 1, 2, 3, 6, 9, 18 and factors of 20 are 1, 2, 4, 5, 10, 20.

(v) **Even number:** The number which are divisible by 2 are called even number. Even numbers are multiples of 2. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 etc are even numbers

(vi) **Odd number:** The number which are not divisible by 2 are called odd number. 1, 3, 5, 7, 9, 11, 13, 15, 17, 19 etc are odd number.

**7. Description of Venn diagram:**



1 → Only A

2 → Only B

3 → Only C

4 → Only A and B

5 → Only A and C

6 → Only B and C

7 → All

8 → Not A, B and C

4,7 → Both A and B or at least A and B

5,7 → Both A and C or at least A and C

6,7 → Both B and C or at least B and C

1,4,5,7 → A

2,4,6,7 → B

3,5,6,7 → C



# Statistics

## (I - A) Pie Chart [Drawing Pie Chart]

1. If  $a$  = number of A,  $b$  = number of B,  $c$  = number of C,  $n$  = total number.

$$\text{angle of A} = \frac{a}{n} \times 360^\circ, \quad \text{angle of B} = \frac{b}{n} \times 360^\circ, \quad \text{angle of C} = \frac{c}{n} \times 360^\circ$$

2. If  $A = a\%$ ,  $B = b\%$ ,  $C = c\%$

$$\text{angle of A} = \frac{a}{100} \times 360^\circ, \quad \text{angle of B} = \frac{b}{100} \times 360^\circ, \quad \text{angle of C} = \frac{c}{100} \times 360^\circ$$

## (i-B) Pie Chart [Calculation From Pie Chart]

1.  $\frac{\text{amount of A}}{\text{amount of B}} = \frac{\text{angle of A}}{\text{angle of B}}$
2.  $\frac{\text{amount of A}}{\text{total amount}} = \frac{\text{angle of A}}{360}$

## (ii) Histogram

### General Information

1. In a histogram frequency is proportional to area. If frequency is  $F$  and area is  $A$ , then  $F = kA$ .
2. The value of  $k$  maybe 1 or any other number. If  $k=1$ , frequency = area.
3. The rectangle of histogram are called bar.

### To find frequency from a given histogram follow the following steps:

1. In these type of question, frequency of one bar is given. Reading the question carefully, find the frequency and area of that bar.
2. Using that frequency and area, and using the formula  $F = kA$ , find the value of  $k$ .
3. Use  $F_1 = kA$ ,  $F_2 = kA_2$ ,  $F_3 = kA_3$ ,  $F_4 = kA_4$  .....to find the unknown frequencies.

Here,

$F_1$  = frequency of 1<sup>st</sup> bar,  $A_1$  = area of 1<sup>st</sup> bar

$F_2$  = frequency of 2<sup>st</sup> bar,  $A_2$  = area of 2<sup>st</sup> bar

$F_3$  = frequency of 3<sup>st</sup> bar,  $A_3$  = area of 3<sup>st</sup> bar

$F_4$  = frequency of 4<sup>st</sup> bar,  $A_4$  = area of 4<sup>st</sup> bar

### To complete an incomplete histogram follow the following steps:

1. In these type of question, one bar must be drawn in the question. Find the frequency and area of that bar.
2. Using that frequency and area, and using the formula  $F=kA$ , find the value of  $k$ .



- Using the formula, frequency density =  $\frac{\text{frequency}}{\text{class width}} \times \frac{1}{k}$ , find the frequency density of the remaining bars.
- Taking frequency density as height and class interval as width, draw the remaining bars.

**To draw a new histogram from a frequency table follow the following steps:**

- Find the frequency density of the bars using the formula.

$$\text{frequency density} = \frac{\text{frequency}}{\text{class width}}, [k = 1]$$

- Taking frequency density as height and class interval as width, draw the bars.
- If all class width of the given frequency table have the same width, the frequency of the table may be taken as frequency density.

### (iii) Bar chart

- Horizontal axis is for name of something and vertical axis for frequency.
- To draw a bar chart, if a scale is given, follow that scale, otherwise choose a suitable scale.
- Widths of all bars of a bar chart are same.

### Comparison of Bar Chart and Histogram

- In a bar chart all bars must have the same width, but in a histogram all bars may have same width or not.
- In a bar chart vertical axis represent frequency, but in a histogram vertical axis represent frequency density.
- In a bar chart name of something is written in horizontal axis, but in a histogram number of class width is written in the horizontal axis.
- In a bar chart, frequency = height of the bar  
In a histogram, frequency = area of the bar or  $k \times$  area of the bar.

### (iv - A) Mean, Median, Mode [From Table]

Mean means average, median means middle number and mode means most occurring frequency.

- Mean:**

(i) The number of in the upper row are the values of number ( $x$ ) and the number in the low row are their frequency ( $f$ ).

$$\text{mean} = \frac{\sum xf}{\sum f} = \frac{\text{sum of } x \times f}{\text{sum of } f}$$

(ii) If  $x$  is given in range, take the midpoint of the range as the value of  $x$ .

$$\text{midpoint} = \frac{\text{starting number} + \text{ending number}}{2}$$



**2. Median**

(a) To find median value of  $x$ , follow the following steps..

(i) Find cumulative frequency .

(ii) Find median frequency using  $\frac{n+1}{2}$ , where  $n$  is total frequency.

(iii) If  $\frac{n+1}{2}$  is decimal number, suppose 21.5, find the  $x$  values corresponding to 21<sup>st</sup> and 22<sup>nd</sup> frequency and take their average.

(b) If  $x$  is given in range, median value of  $x$  cannot be found, only median interval (range) can be found. To find median interval, find median frequency. The interval where median frequency lies is the median interval.

**3. Mode:**

The value of  $x$  which has the highest frequency is the mode.

**(iv-B) Mean, Median, Mode [Other]**

1. 
$$\text{mean} = \frac{\text{sum of numbers}}{\text{number of numbers}}$$

**2. Median**

To find median from raw data, follow the following steps.

(i) Arrange the numbers in ascending order.

(ii) Find the median using  $\frac{n+1}{2}$ , where  $n$  is total frequency.

(iii) If  $\frac{n+1}{2}$  is a whole number, find the  $x$  value corresponding to that frequency.

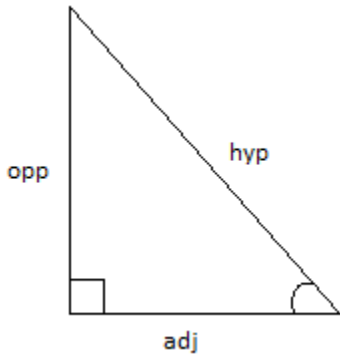
(iv) If  $\frac{n+1}{2}$  is a decimal number, suppose 3.5, find the average of 3<sup>rd</sup> and 4<sup>th</sup> number. This average is the median of  $x$ .

3. The number which is given the most time is mode.

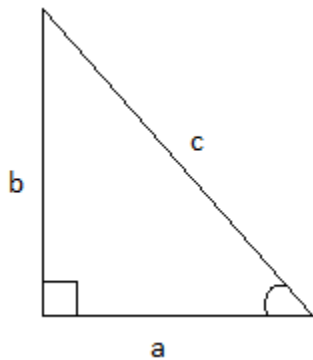
# Trigonometry

## Two dimension

- (i) The side opposite the right angle triangle is called hypotenuse (hyp).  
 (ii) The side opposite the marked angle is called Opposite (opp).  
 (iii) The other side is called Adjacent (adj).

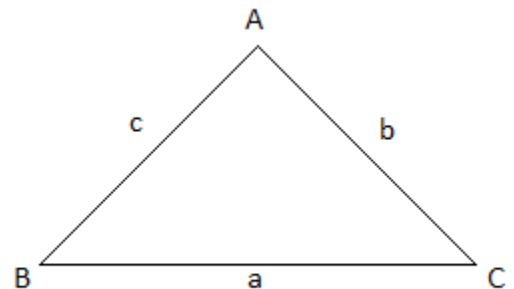


- (i)  $\sin x = \frac{opp}{hyp}$   
 (ii)  $\cos x = \frac{adj}{hyp}$   
 (iii)  $\tan x = \frac{opp}{adj}$
- Pythagoras Theorem:** In an right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.



$$a^2 + b^2 = c^2$$

- Sine rule:**  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
- Cosine rule:**
  - To find side,  $a^2 = b^2 + c^2 - 2bc \cos A$
  - To find angle,  $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$





## Use of Trigonometric Formulae

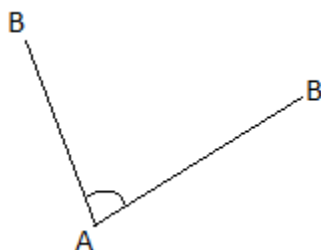
1. In a right-angled triangle, if one acute angle and one side known, to find the other two sides, *sin*, *cos* or *tan* can be used.
2. In a right-angled triangle, if two sides are known, to find the acute angles *sin*, *cos* or *tan* can be used.
3. In a right-angled triangle, if two sides are known but no acute angle is known, to find the third side **Pythagoras Theorem** can be used.
4. In any triangle (right-angled or not), if two sides and included angle are known, to find the third side **Cosine rule** can be used.
5. In any triangle, if three sides are known, to find angles **Cosine rule** can be used.
6. In any triangle, if two sides and one of their opposite angles are known, to find their other opposite angle Sine Rules can be used.
7. In any triangle, if two angles and one of their opposite sides are known to find the other opposite side Sine Rules can be used.

## Three Dimension

1. The angle between a vertical line and a horizontal line must be  $90^\circ$ , no matter it looks like  $90^\circ$  or not.
2. The angles of a square or a rectangle must be  $90^\circ$ .
3. **Angle between a line and a plane:** If a line PO intersects a given plane at O and PN is the perpendicular from P to the plane. The angle PON is defined as the angle between the line and the plane.
4. **Angle between two planes:** Two planes which are not parallel intersect in a straight line. Draw two lines, one in each plane and each perpendicular to the common line of intersection. The angle between these two lines is defined as the angle between the planes.

## Bearing

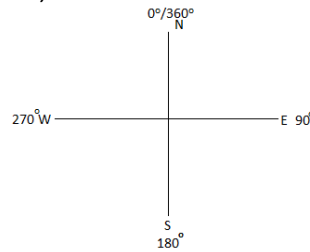
1. The clockwise angle between AB and due north line AN at A is defines as the bearing of B from A.  $\angle BAN$  is the bearing of B from A.



- The clockwise angle between AB and due north line BN at B is defined as the bearing of A from B. Reflex  $\angle ABN$  is the bearing of A from B.



- If point A lies above the horizontal and you need bearing of A, find the foot F of A. The bearing of F will be taken as bearing of A.
- Bearing of due north line is  $0^{\circ}/360^{\circ}$   
 Bearing of due east line is  $90^{\circ}$   
 Bearing of due south line is  $180^{\circ}$   
 Bearing of due west line is  $270^{\circ}$



### Angle of elevation and depression

- The angle between AB and the projection AC, is defined as the angle of elevation of B from A.  $\angle BAC$  is the angle of elevation of B from A.
- The angle between AB and the horizontal line BC, is defined as the angle of depression of A from B.  $\angle ABC$  is the angle of depression of A from B.

### Trigonometry Equation

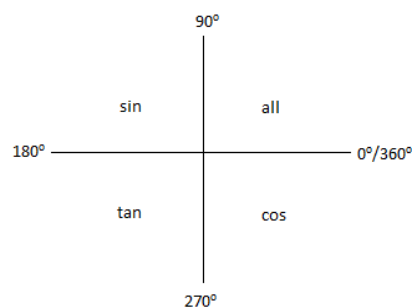
- While solving trigonometric equation, by using calculator sometimes you get values (1<sup>st</sup> value) which is not in the given range. In that case you need to find the 2<sup>nd</sup> value. To find the 2<sup>nd</sup> value, follow the rules given in the table.

	sin	tan	Cos
1 <sup>st</sup> value	From calculator	From calculator	From calculator



2 <sup>nd</sup> value	$180^\circ - 1^{\text{st}}$ value	$180^\circ + 1^{\text{st}}$ value	$360^\circ - 1^{\text{st}}$ value
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2. In the 1<sup>st</sup> quadrant, all positive.  
 In the 2<sup>nd</sup> quadrant sin positive.  
 In the 3<sup>rd</sup> quadrant tan positive.  
 In the 4<sup>th</sup> quadrant cos positive.



3. Acute angle = between  $0^\circ$  and  $90^\circ$   
 Obtuse angle = between  $90^\circ$  and  $180^\circ$   
 Reflex angle = between  $180^\circ$  and  $360^\circ$

# Mensuration

## 1. Sector

(i) Area  $A = \frac{\theta}{360} \times \pi r^2$

(ii) Arc length  $l = \frac{\theta}{360} \times 2\pi r$

(iii) Perimeter  $= \frac{\theta}{360} \times 2\pi r + 2r$

(iv) Segment area = Sector area – Triangle area.

## 2. Circle

(i) Area =  $\pi r^2$

(ii) Circumference =  $2\pi r$

## 3. Trapezium

Area of Trapezium =  $\frac{1}{2}(a + b)h$  [a and b are the length of parallel sides, h is the distance of parallel sides]

## 4. Rhombus

(i) Area of Rhombus =  $(\frac{1}{2} \times d_1 \times d_2)$  or  $(base \times height)$  or  $(ab \sin \theta)$  [ $d_1$  &  $d_2$  are two diagonals of the rhombus.]

(ii) length of 1 side of a Rhombus =  $\sqrt{\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2}$

(iii) The diagonals of a Rhombus bisect each other at right angle.

## 5. Parallelogram

Area of Parallelogram =  $(base \times height)$  or  $(ab \sin \theta)$

## 6. Triangle

Area of Triangle =  $(\frac{1}{2} \times base \times height)$  or  $(\frac{1}{2} ab \sin \theta)$  [a and b are two sides of the triangle.  $\theta$  is the angle made by two sides]

## 7. Rectangle

(i) Area = length x breadth

(ii) Perimeter =  $2l + 2b$  or  $2(l + b)$

## 8. Square

(i) Area = length<sup>2</sup>

(ii) Perimeter =  $4L$

## 9. Cone

(i) Volume =  $\frac{1}{3} \pi r^2 h$

(ii) Curved surface area =  $\pi r l$

(iii) Total surface area =  $\pi r l + \pi r^2$

(iv)  $l^2 = h^2 + r^2$  [ $l$  = slant height and  $h$  = perpendicular height of the cone]

(v) When a cone is made by joining two straight edges of a sector,

*circumference of the base of the cone = arc length of the sector.*

*slant height of the cone = radius of the sector*



**10. Cylinder**

- (i) Volume =  $\pi r^2 h$
- (ii) Curved surface area =  $2\pi r h$
- (iii) Total surface area (closed solid with lid) =  $2\pi r h + 2\pi r^2$
- (iv) Total surface area(no lid) =  $2\pi r h + \pi r^2$
- (v) Volumes of material of a hollow cylinder = External volume – Internal volume.

**11. Sphere**

- (i) Volume =  $\frac{4}{3}\pi r^3$
- (ii) Surface area =  $4\pi r^2$
- (iii) Volume of material of a hollow sphere = External volume – Internal volume.

**12. Hemisphere**

- (i) Volume =  $\frac{2}{3}\pi r^3$
- (ii) Curved surface area =  $2\pi r^2$
- (iii) Total surface area =  $3\pi r^2$

**13. Pyramid**

$$\text{Volume of Pyramid} = \frac{1}{3} \times \text{base area} \times \text{height}$$

**14. Cuboid (box)**

- (i) Volume of Cuboid = length x breadth x height
- (ii) Surface area of closed box =  $2lb + 2lh + 2bh$
- (iii) Surface area of open box =  $lb + 2lh + 2bh$
- (iv)  $d^2 = l^2 + b^2 + h^2$  [d = diagonal, l = length, b = breadth, h = height]

**15. Cube**

$$\text{Volume of Cube} = (\text{length})^3$$

**16. Prism**

$$\text{Volume of Prism} = \text{Area of cross-section} \times \text{length}$$

**17. (i) Volume of water discharged through a pipe in 1 minute =  $A \times S \times 60$** 

$$\text{(ii) Time required to fill a container by a pipe} = \frac{\text{volume of container}}{\text{rate of flow}}$$

18. When some smaller objects are made from a bigger object,  
volume of smaller objects = volume of bigger object.

**19. Kite**

$$\text{Area of Kite} = \frac{1}{2} \times d_1 \times d_2$$

20. (i) If a number is  $x$ , to the nearest metre, maximum value of the number =  $x + 0.5$   
minimum value of the number =  $x - 0.5$

**21. Differentiation for Mensuration**

$$\text{General rule: } \frac{d}{dx}(x^n) = nx^{n-1}$$

# Equation Graph

## Paper 1

### (I) Calculus

#### 1. Differentiation

- $\frac{d}{dx}(\text{only number}) = 0$ ,  $\frac{d}{dx}(x) = 1$ ,  $\frac{d}{dx}(x^2) = 2x$ ,  $\frac{d}{dx}(x^3) = 3x^2$
- General rule:  $\frac{d}{dx}(x^n) = nx^{n-1}$

#### 2. Indices rule needed for Differentiation

- $\frac{1}{x^n} = x^{-n}$

3.  $\frac{d}{dx}$  means gradient. To find gradient of the curve put the  $x$  value in the  $\frac{dy}{dx}$  expression.

4. At maximum or minimum point,  $\frac{d}{dx} = 0$

5. To find  $x$  value for which  $y$  is maximum or minimum, follow the following steps.

- Find  $\frac{dy}{dx}$
- Taking  $\frac{dy}{dx} = 0$ , find the value of  $x$ .
- Put the  $x$  value in the  $y$  expression.

6. To find the maximum or minimum value of  $y$ , follow the following steps.

- Find  $\frac{dy}{dx}$
- Taking  $\frac{dy}{dx} = 0$ , find the value of  $x$
- Put the  $x$  value in the  $y$  expression.

7. To find maximum or minimum value of  $y$ , follow the following steps.

- Find  $\frac{dy}{dx}$
- Taking  $\frac{dy}{dx} = 0$ , find the value of  $x$
- Put the  $x$  value in the  $y$  expression.

8. To find whether it is maximum or minimum, follow the following steps.



1. Find  $\frac{d^2y}{dx^2}$
2. Taking  $\frac{dy}{dx} = 0$  in  $\frac{d^2y}{dx^2}$ , find the values of  $x$
3. If  $\frac{d^2y}{dx^2}$  is positive, it is minimum and if  $\frac{d^2y}{dx^2}$  is negative it is maximum.

9. Other names of maximum and minimum point are turning point or stationary point.

## (ii) Coordinate Geometry

1. On  $x$ -axis,  $y=0$  and on  $y$ -axis,  $x=0$
2. If  $A(x_1y_1)$  and  $B(x_2y_2)$ , gradient of AB,  $m = \frac{y_2 - y_1}{x_2 - x_1}$
3. If  $A(x_1y_1)$  and  $B(x_2y_2)$ , length of AB =  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
4. If  $A(x_1y_1)$  and  $B(x_2y_2)$ , midpoint of AB =  $(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2})$
5. If  $A(x_1y_1)$  and  $B(x_2y_2)$ , equation of AB:  $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$
6. **(i) When  $x$ -coordinate of two points are equal:** If  $A(p, q)$  and  $B(p, r)$ , equation of AB:  $x=p$   
**(ii) When  $y$ -coordinate of two points are equal:** If  $A(p, q)$  and  $B(x, q)$ , equation of AB:  $y=q$
7. The equation of a straight line with gradient  $m$  and passing through the point  $(x_1, y_1)$  is  

$$y - y_1 = m(x - x_1)$$
8. If  $y = mx + c$ ,  $m$  is the gradient of the line.
9. If the point  $(x_1, y_1)$  satisfy the line  $y = mx + c$ , the line passes through the point.
10. To find the coordinates of the intersecting point of the two lines, solve the two equations representing the lines. The  $x$  and  $y$  value are the coordinates of the intersecting point.
11. Coordinate of Origin, is  $(0, 0)$ .

## (iii) Distance-Time Graph

1. speed = gradient of distance-time graph.
2. If speed is constant distance-time graph becomes a straight line, otherwise curve line.

## (iv) Speed-Time Graph

1. acceleration = gradient of speed-time graph.
2. distance travelled = area under graph.
3. If acceleration is constant, speed-time graph becomes a straight line, otherwise curve line.

## (v) Inequality Graph

### 1. Name of inequality symbols:

- i. less than ( $<$ ).
- ii. less than or equal to ( $\leq$ )

- iii. greater than ( $>$ )
- iv. greater than or equal to ( $\geq$ )

## 2. Finding inequality when inequality graph is given:

### For slant and horizontal line:

- i. (a) If shaded area is below the line, replace = sign of the line equation by  $<$ . [If boundary is not mentioned]  
(b) If shaded area is above the line, replace = sign of the line equation by  $>$ . [If boundary is not mentioned]
- ii. (a) If shaded area is below the line, replace = sign of the line equation by  $\leq$ . [If boundary is not mentioned]  
(b) If shaded area is above the line, replace = sign of the line equation by  $\geq$ . [If boundary is not mentioned]

### For vertical line:

- i. (a) If shaded area is left-hand side of the line, replace = sign of the line equation by  $<$ . [If boundary is not mentioned]  
(b) If shaded area is right-hand side of the line, replace = sign of the line equation by  $>$ . [If boundary is not mentioned]
- ii. (a) If shaded area is left-hand side of the line, replace = sign of the line equation by  $\leq$ . [If boundary is not mentioned]  
(b) If shaded area is right-hand side of the line, replace = sign of the line equation by  $\geq$ . [If boundary is not mentioned]

## 3. Shading the defined region:

- i. If three inequality is given, find the triangle made by the three lines and shade the triangle.
- ii. If four inequality is given, find the quadrilateral made by the four lines and shade the quadrilateral.

## Paper 2

### 1. Completing table

- i. Until the year 1989 tables were given with 2 rows only, top row for  $x$  and bottom row for  $y$ . For this kind of table, write the given equation in the calculator and use the CALC button of the calculator to get the missing values of  $y$ .
- ii. From the year 1990, tables are given with more than 2 rows. Top row for  $x$ , bottom row for  $y$  and middle rows for each term of the equation. For this kind of table, first fill up the empty boxes and middle rows for each term of the equation. For this kind of table, first fill up the empty boxes of the middle rows manually or by using CALC button of the calculator. Then add or



multiply the numbers in the middle rows according to the equation to get the missing  $y$  values in the bottom row.

## 2. Drawing $x$ -axis and $y$ -axis

Some students draw  $x$  and  $y$  axis randomly. Later when they write the  $x$  and  $y$  values on the axes, sometimes they don't get sufficient space to write the values. As a result they need to rub their working and do it in a graph paper. To avoid this kind of situation, it is better to draw the axes after some measurement.

- i. To draw the vertical axis ( $x$  axis), look at the top row of the table to find the largest value of positive  $x$  and negative  $x$ . Then look at the scale of the  $x$  axis in the question paper. Calculate how many cm of space you need to write  $x$ -values. Normally in a graph paper, horizontal space is 20 cm. Try to leave extra space from both side equally, but do not make it decimal. Mark a point which divide the spaces for negative and positive  $x$  values. Draw a vertical line ( $y$ -axis ) at that point.
- ii. To draw the horizontal axis ( $y$  axis), look at the top row of the table to find the largest value of positive  $y$  and negative  $y$ . Then look at the scale of the  $y$  axis in the question paper. Calculate how many cm of space you need to write  $y$ -values. Normally in a graph paper, horizontal space is 25 cm. Try to leave extra space from both side equally, but do not make it decimal. Mark a point which divide the spaces for negative and positive  $x$  values. Draw a vertical line ( $y$ -axis ) at that point.

## 3. Plotting the points on the graph paper

If  $x$  and  $y$  values are whole number, then it is easy to plot the points. But if they are decimal numbers, some students do not understand where will be the position of the point. The following method can be applied to identify the position  $x$  and  $y$ .

- i. For the position of  $x$ , look at the given scale for  $x$  in the question and find 1 unit is how many cm. Multiply the value of  $x$  by this cm. From the origin go rightward if  $x$  is positive and go leftward if  $x$  is negative with the multiplied cm of distance which is the position of  $x$ .
- ii. For the position of  $y$ , look at the given scale for  $y$  in the question and find 1 unit is how many cm. Multiply the value of  $y$  by this cm. From the position of  $x$  go upward if  $y$  is positive and go downward if  $y$  is negative with the multiplied cm of distance which is the position of the point.
- iii. After plotting the points, join them in free hand to make to smooth curve. If you notice that one point is breaking the smoothness of the curve, check if there is any mistake to calculate or plot the point.

## 4. Finding gradient of a curve

There are following two method to find the gradient of a curve at a particular point. If method is mentioned in the question, you must find the gradient in that way. Otherwise you will use calculus method, since it gives the accurate answer.



- i. Calculus method: Find  $\frac{dy}{dx}$  and put the given  $x$  value in the expression of  $\frac{dy}{dx}$ .
- ii. Graphical method:
  - (a) Find the point on the curve for the given  $x$  value.
  - (b) Draw a tangent on the curve at that point.
  - (c) Mark any two suitable points on the tangent and write the coordinates of these points in your answer book. You can take the point where tangent is drawn as one of these two points.
  - (d) Using the formula  $m = \frac{y_2 - y_1}{x_2 - x_1}$  calculate the gradient of the curve.

### 5. Finding the equation of a line to solve an equation

To find the equation of a line to solve an equation follow the following steps:

- i. Copy the given equation which you need to solve.
- ii. Observe the left hand side of the equation. It may be exactly same as  $y$  equation or not. If it is exactly same as  $y$ , in the second line write  $y = \text{right hand side number}$ , which is the equation of the line.
- iii. If it is not exactly same as  $y$ , follow the following steps:
  - (a) In the second line, write the expression you need to bring  $y$  and add or subtract something to balance this line with previous line keeping the right hand side name.
  - (b) In the third line write  $y$  in place of  $y$  expression and take the other terms in the right hand side.
  - (c) Simplify the right hand side if necessary.

### 6. Solving an equation

There are two method to solve an equation: *algebraic* and *graphical*. In the algebraic method we use middle term break or  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  to solve a quadratic equation and factor theorem to solve a cubic equation. But in graph chapter, in 99% cases you will be asked to use your graph to solve a given equation. For graphical method follow the following steps.

- i. Copy the given equation in your answer book.
- ii. Find the line you need to solve the equation.
- iii. If the line is not drawn, draw the line on the graph paper.
- iv. To draw the line in the form  $x = a$ , where  $a$  is a number, draw a vertical line at  $x = a$ . To draw the line in form of  $y = b$ , where  $b$  is a number, draw a horizontal at  $y = b$ . To draw the line in the form of  $y = mx + c$ , make a small table with 2 row and 4 column. In the first column write  $x$  and  $y$  and in the other column write 3 suitable values of  $x$  and corresponding 3 values of  $y$  which you have to calculate from the equation  $y = mx + c$ . Plot these 3 points on the graph paper and join them to scale to get the line.
- v. Mark the intersecting point(s) between the curve and the line. Draw perpendicular(s) on the  $x$ -axis from intersecting point(s).
- vi. Find the  $x$  value(s) where the perpendicular(s) meet the  $x$ -axis which is (arc) the solution of the given equation.

- vii. If you face problem to find  $x$  value, count the distance of that meeting point from the origin and also calculate from the given scale of  $x$ , 1 cm is how many unit. Multiply that distance with this unit to get  $x$  value.

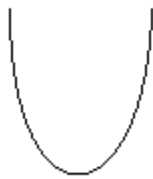
## 7. Shape of graph

### i. Quadratic Graph

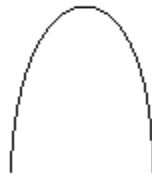
(a) Equation of a quadratic graph is  $y = ax^2 + bx + c$

(b) The shape of a quadratic graph is:

(i) if  $a > 0$



(ii) if  $a < 0$

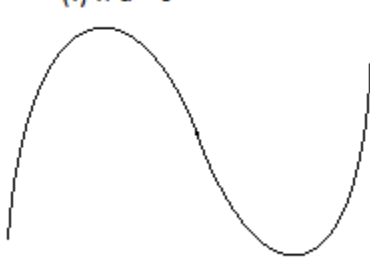


### ii. Cubic Graph

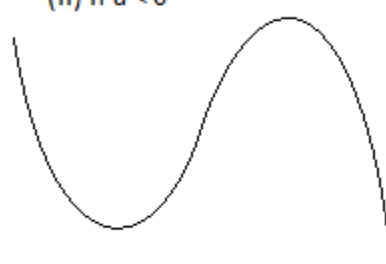
(a) Equation of a cubic graph is  $y = ax^3 + bx^2 + cx + d$

(b) The shape of cubic graph is:

(i) if  $a > 0$



(ii) if  $a < 0$



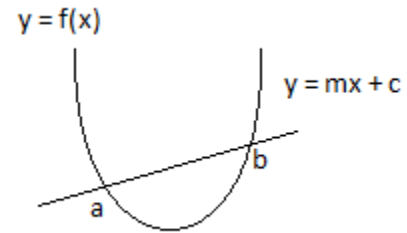
## 8. Solving an inequality from the graph

To solve an inequality by graphical method, follow the following steps:

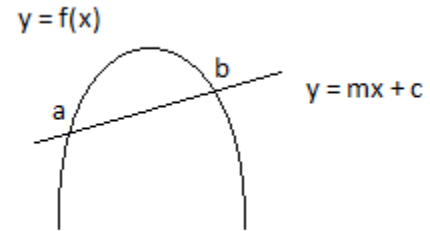
- i. Copy the given inequality in your answer book.
- ii. Find the line you need to solve the inequality.
- iii. If the line is not drawn, draw the line on the graph paper.
- iv. Mark the intersecting points between the curve and the line. Draw perpendiculars on the  $x$ -axis from intersecting points.
- v. Find the  $x$  values where the perpendiculars meet the  $x$ -axis.

vi. If it is a quadratic inequality, there will be two  $x$  values, suppose  $a$  and  $b$  where  $a < b$ .

For  $f(x) > mx + c$ , answer is  $x < a, x > b$   
 For  $f(x) < mx + c$ , answer is  $a < x < b$

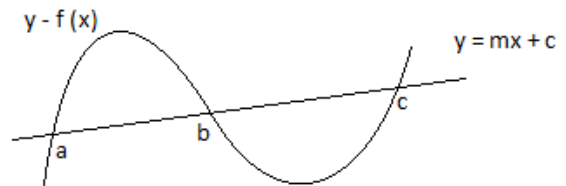


For  $f(x) > mx + c$ , answer is  $a < x < b$   
 For  $f(x) < mx + c$ , answer is  $x < a, x > b$

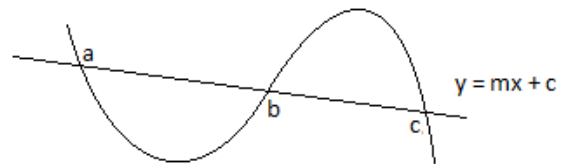


vii. If it is a cubic inequality, there will be three  $x$  values, suppose  $a, b$  and  $c$  where  $x < b < c$ .

For  $f(x) > mx + c$ , answer is  $a < x < b, x > c$   
 For  $f(x) < mx + c$ , answer is  $x < a, b < x < c$



For  $f(x) > mx + c$ , answer is  $a < x < b, x < c$   
 For  $f(x) < mx + c$ , answer is  $a < x < b, x > c$



# Arithmetic

## (i) Ratio

- If any amount  $A$  is divided between  $P$ ,  $Q$  and  $R$  in the ratio  $p:q:r$   

$$\text{share of } P = \frac{p}{p+q+r} \times A, \text{ share of } Q = \frac{q}{p+q+r} \times A, \text{ share of } R = \frac{r}{p+q+r} \times A$$
- $$\frac{\text{share of } P}{\text{share of } Q} = \frac{\text{ratio of } P}{\text{ratio of } Q}$$
- $$\frac{\text{share of } P}{\text{total amount}} = \frac{\text{ratio of } P}{\text{total ratio}}$$
- If  $A:B = p:q$  and  $B:C = r:s$ ,  $A : B : C = pr : qr : qs$

## (ii) Percentage

- Profit = sale price – cost price
- Loss = cost price – sale price
- Percentage profit =  $\frac{\text{profit}}{\text{cost price}} \times 100$
- Percentage loss =  $\frac{\text{loss}}{\text{cost price}} \times 100$
- Percentage increase =  $\frac{\text{increase}}{\text{original value}} \times 100$
- If  $a$  = number of  $A$ ,  $b$  = number of  $B$ ,  $c$  = number of  $C$ ,  $n$  = total number.  

$$\% \text{ of } A = \frac{a}{n} \times 100, \quad \% \text{ of } B = \frac{b}{n} \times 100, \quad \% \text{ of } C = \frac{c}{n} \times 100$$
- % of  $A$  compared with  $B = \frac{A}{B} \times 100$

## (iii) Speed, Distance, Time

- distance = speed  $\times$  time
- If a number is  $x$ , to the nearest 10, maximum value of the number =  $x + 5$   
 minimum value of the number =  $x - 5$
- If distance is constant, least time =  $\frac{\text{distance}}{\text{maximum speed}}$
- If there are two parts of the journey and  $S_1$  = speed of 1<sup>st</sup> part,  $d_1$  = distance of 1<sup>st</sup> part,  $t_1$  = time of 1<sup>st</sup> part,  $s_2$  = speed of 2<sup>nd</sup> part,  $d_2$  = distance of 2<sup>nd</sup> part,  $t_2$  = time of 2<sup>nd</sup> part,  $s$  = average speed of the whole journey,  $d$  = distance of whole journey,  $t$  = time of whole journey.

$$s = \frac{d}{t} = \frac{d_1 + d_2}{t_1 + t_2} = \frac{(s_1 \times t_1) + (s_2 \times t_2)}{\left(\frac{d_1}{s_1}\right) + \left(\frac{d_2}{s_2}\right)}$$

## (iv) Map Scale

- Scale of a map = (map distance) : (actual distance).
- Scale of a map is written in the form  $1 : n$ , where 1 is map distance ratio and  $n$  is actual distance ratio.



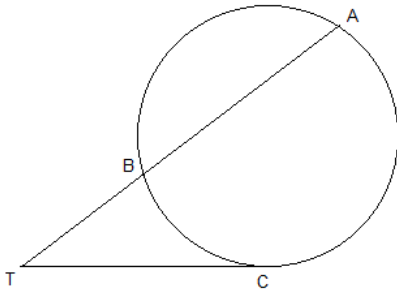
3.  $\frac{d_1}{d_2} = \frac{1}{n}$ , where  $d_1$  = map distance and  $d_2$  = actual distance.
4.  $\frac{A_1}{A_2} = \left(\frac{1}{n}\right)^2$ , where  $A_1$  = map area and  $A_2$  = actual area.

# Geometry

## (i) Circle Theorems

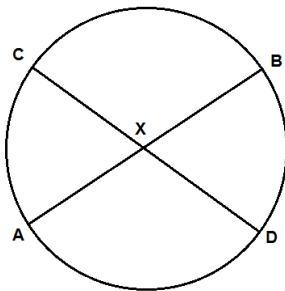
1. Secant-tangent theorem: If, from any point outside a circle, a secant and a tangent are drawn, the rectangle contained by the whole secant and the part of it outside the circle is equal to the square on the tangent.

$$AT \times BT = CT^2$$



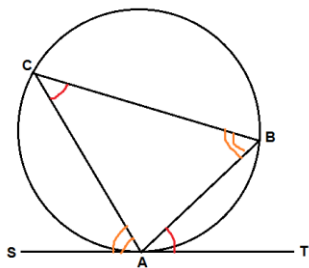
2. Intersecting chord theorem: If two chords of a circle intersect either inside or outside the circle, the product of the segments of one chord is equal to the product of the segments of the other chord.

$$AX \times BX = CX \times DX$$



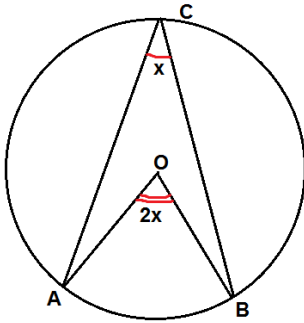
3. Alternate segment theorem: The angle between a tangent and a chord through the point of contact is equal to the angle subtended by the chord in the alternate segment.

$$\angle TAB = \angle BCA \text{ and } \angle SAC = \angle CBA$$



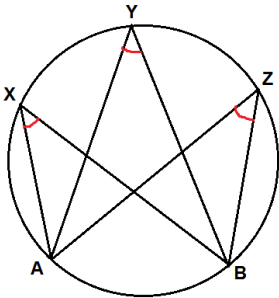
4. The angle subtended at the centre of a circle is twice the angle subtended at the circumference.

$$\angle AOB = 2 \angle ACB$$



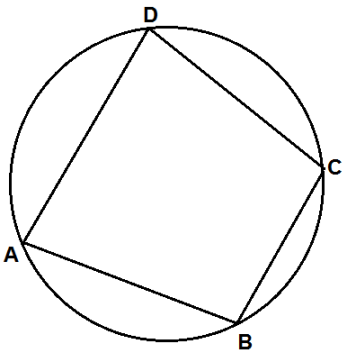
5. Angle subtended by an arc in the same segment of a circle is equal.

$$\angle AXB = \angle AYB = \angle AZB$$



6. The opposite angles in a cyclic quadrilateral add up to  $180^\circ$ .

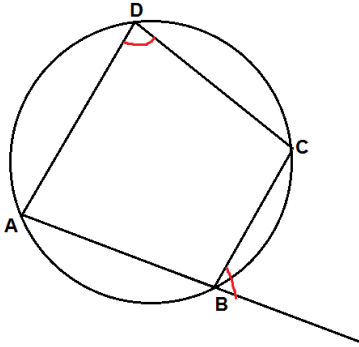
$$\angle A + \angle C = 180^\circ, \angle B + \angle D = 180^\circ$$





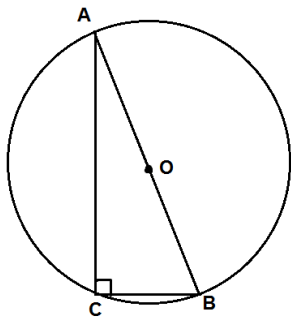
7. The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

$$\angle CBE = \angle ADC$$



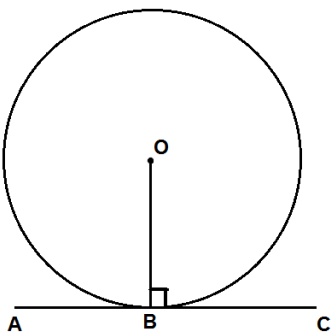
8. The angle in a semi-circle is a right angle.

$$\angle ACB = 90^\circ$$



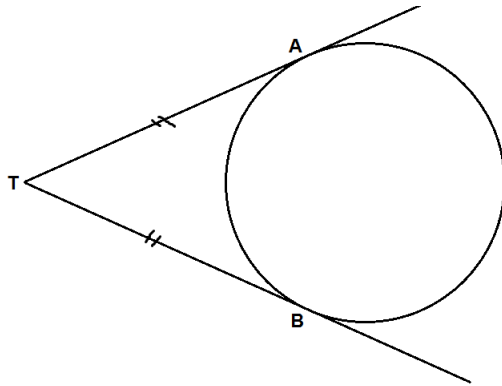
9. The angle between a tangent and the radius drawn to the point of contact is  $90^\circ$ .

$$\angle ABO = 90^\circ$$



10. From any point outside a circle just two tangents to the circle may be drawn and they are of equal length.

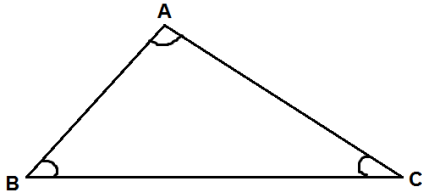
$TA = TB$



**(ii) Triangle / Parallel Line**

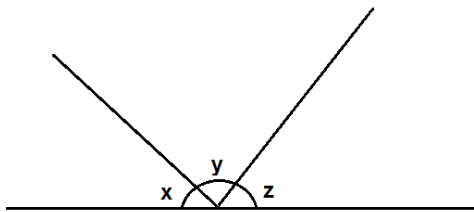
1. The angle sum of a triangle is  $180^\circ$ .

$$\angle A + \angle B + \angle C = 180^\circ$$



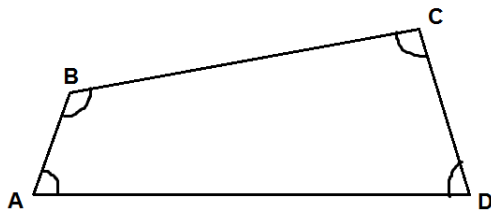
2. The angles on a straight line add up to  $180^\circ$

$$x + y + z = 180^\circ$$

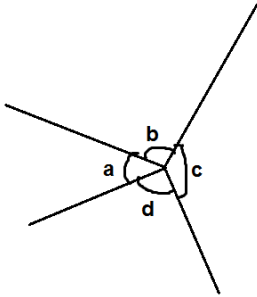


3. The angle sum of a quadrilateral is  $360^\circ$ .

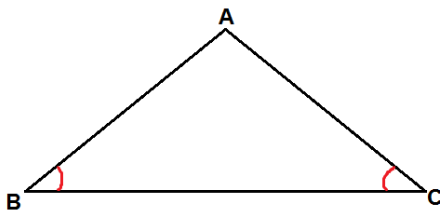
$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$



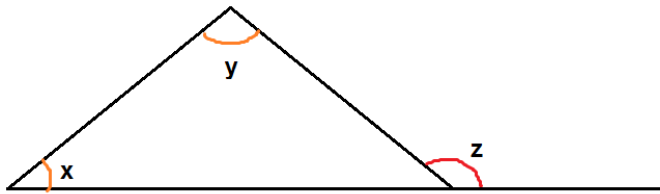
4. The angles at a point add up to  $360^\circ$ .  
 $a + b + c + d = 360^\circ$



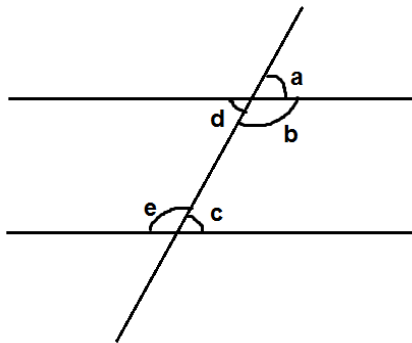
5. If two sides of a triangle are equal, their opposite angles are also equal.  
 If  $AB = AC$ ,  $\angle B = \angle C$



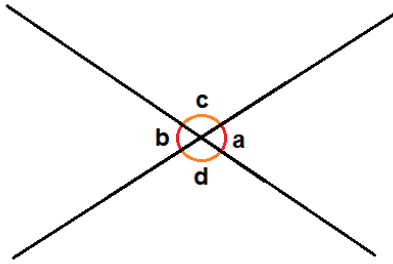
6. The exterior angle of a triangle is equal to the sum of the two interior opposite angles.  
 $z = x + y$



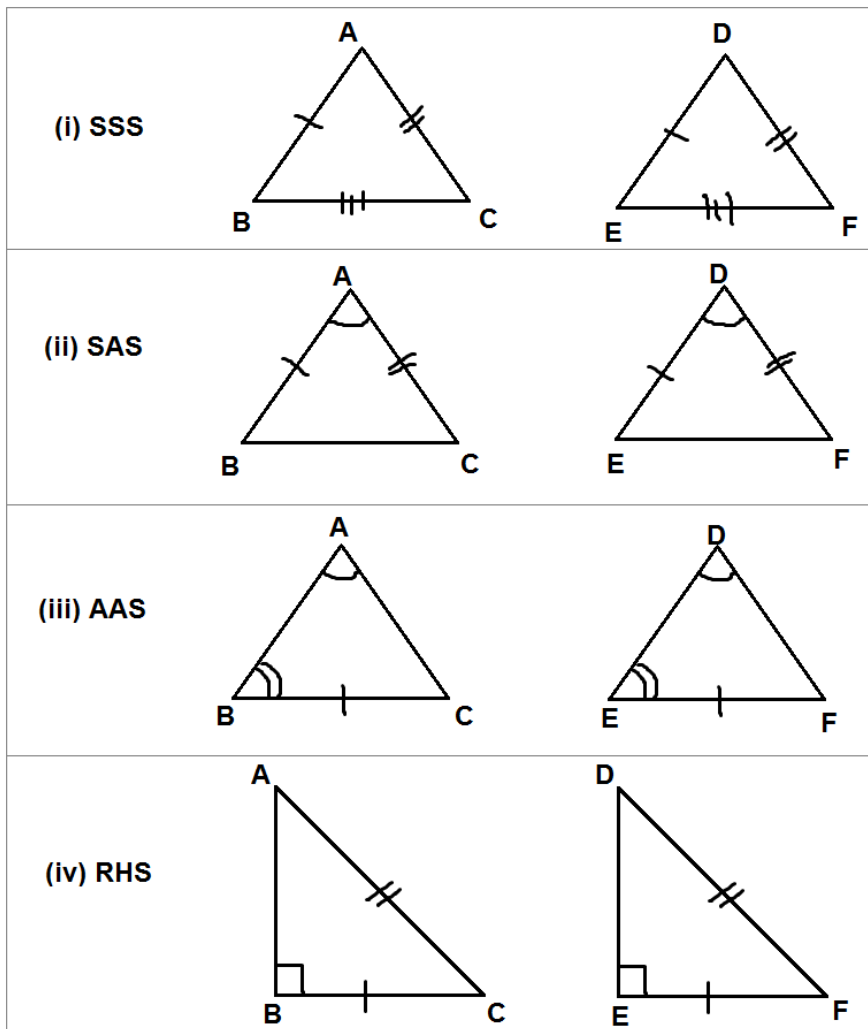
7. If two lines are parallel,  
 (i) corresponding angles are equal,  $a = c$   
 (ii) alternate angles are equal,  $c = d$ ,  $b = e$   
 (iii) sum of allied angles is  $180^\circ$ .  $b + c = 180^\circ$ ,  $d + e = 180^\circ$



8. If two lines intersect, vertically opposite angles are equal.  
 $a = b, c = d$



9. Congruency triangle: Two triangle will be congruent (equal in all respect) if they satisfy any one of the following four condition.



**(iii) Polygon**

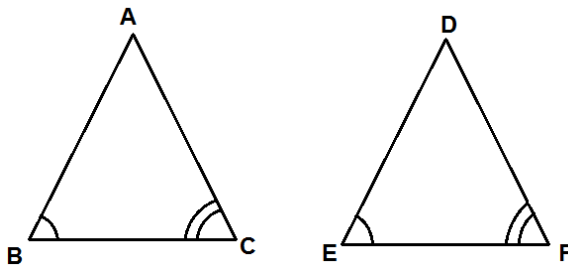
- Number of sides of a regular polygon =  $\frac{360}{\text{exterior angle}}$
- For any polygon, interior angle + exterior angle =  $180^\circ$ .

- One interior angle of a regular polygon =  $\frac{(2n-4) \times 90^\circ}{n}$ .
- Sum of interior angles of any pentagon =  $540^\circ$  and any hexagon =  $720^\circ$ .
- Sum of exterior angle of any polygon =  $360^\circ$ .
- Area of a regular polygon = area of 1 triangle x n, where n is number of sides.
- Sum of interior angle of any polygon =  $(2n - 4) \times 90^\circ$ , where n is number of sides.

**(iv) Similarity**

- Two triangles are similar if they have the same angles.
- Opposite sides of equal angles of similar triangles are called corresponding sides.
- If two triangles are similar, their corresponding sides are proportional.

If  $\triangle ABC$  and  $\triangle DEF$  are similar,  $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$



- Side ratio, length ratio, breadth ratio, radius ratio, diameter ratio, circumference ratio, height ratio etc. are called linear ratio. Remember, the word 'linear' came from the word 'line'.
- $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$ , where  $\frac{A_1}{A_2}$  = area ratio and  $\left(\frac{l_1}{l_2}\right)$  = linear ratio.
- $\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$ , where  $\frac{V_1}{V_2}$  = volume ratio and  $\left(\frac{l_1}{l_2}\right)$  = linear ratio.

**(v) Construction / Locus**

- Locus of points equidistant from A and B = Perpendicular bisector of AB.
- Locus of points equidistant from AB and BC = Bisector of  $\angle ABC$ .
- Locus of points having constant distance from AB = Parallel line above or below AB with given distance.
- Locus of points having constant distance from A = A circle with centre A and radius equal to given distance.
- Locus of points X such that  $\angle AXB = 90^\circ$  = A semicircle with AB as diameter.
- Locus of X such that  $\angle ABX = 90^\circ$  = A perpendicular line to AB at B.

**(vi) Symmetry**

- Line symmetry = If an object is folded along a line and one part coincide with another part, then the line is a line symmetry of the object.

2. Rational symmetry = If an object is rotated  $360^\circ$  through a suitable centre, the number of times it covers the same place is equal to number of rational symmetry.



# Algebra

## Indices

1.  $A^m \times a^n = a^{m+n}$
2.  $a^m / a^n = a^{m-n}$
3.  $(a^m)^n = a^{mn}$
4.  $a^0 = 1$
5.  $a^{-n} = \frac{1}{a^n}$
6.  $a^{\frac{1}{n}} = \sqrt[n]{a}$
7.  $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

## Quadratic Equation

1. If  $ax^2 + bx + c = 0$ ,  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

## Inequality

1. Name of inequality symbols:
  - (i) less than (<)
  - (ii) greater than (>)
  - (iii) less than or equal to ( $\leq$ )
  - (iv) greater than or equal to ( $\geq$ )
2. When we multiply or divide by a negative number, the inequality is reversed.
3. If  $x > a$  and  $x$  is an integer, immediate right-hand side integer of  $a$  is the smallest value of  $x$ .
4. If  $x < b$  and  $x$  is an integer, immediate left-hand side integer of  $b$  is the largest value of  $x$ .
5. If  $x > a$ ,  $x < b$  and  $x$  is an integer, take the whole number  $l$  between  $a$  and  $b$ .
6.  $x > a$  and  $x < b$  can be combined as  $a < x < b$ .
7. Integer number: The numbers which are not decimal or fraction are called integer number. Integer numbers are always whole number. Positive integers are 1, 2, 3, 4... Negative integers are -1, -2, -3, -4... The number 0 is also considered as integer.

## Variation

1.  $y$  varies as  $x$  or  $y$  varies directly as  $x$  or  $y$  is proportional to  $x$ . All three statement have the same meaning for these,  $y = kx$ .
2.  $y$  varies inversely as  $x$  or  $y$  is inversely proportional to  $x$ , both the statement have the same meaning and for these,  $y = \frac{k}{x}$ .

## Factor / Remainder Theorem

1. If  $(x + a)$  is a factor of  $f(x)$ ,  $f(-a) = 0$ .



- If  $f(x)$  is divided by  $(x + a)$ , remainder is  $f(-a)$ .

### Prime Factor, LCM, HCF

- For LCM, take all factors and highest power.
- For HCF, take only common factor and lowest power.

### Factorisation

- $a^2 - b^2 = (a - b)(a + b)$

### Evaluation

- The number  $a \times 10^n$  is in standard form when  $1 \leq a < 10$  and  $n$  is positive or negative integer.
- Number facts:
  - Rational number: Fraction numbers, whole numbers and decimal numbers which can be written in fraction are called rational number.  $\frac{5}{17}, \frac{9}{13}, 25.67, 0.25, 0.375$  etc are rational number.
  - Irrational number: The numbers which are not rational number are called irrational number.  $\pi, \sqrt{2}, \sqrt{5}$  etc are irrational numbers.

### Sequence

- For arithmetic progression (A.P.),  $T_n = a + (n - 1)d$
- For  $T_n$  is given, to find  $T_1, T_2, T_3, T_4, T_5, \dots$ , put  $n = 1, 2, 3, 4, 5$  in the  $T_n$  expression.